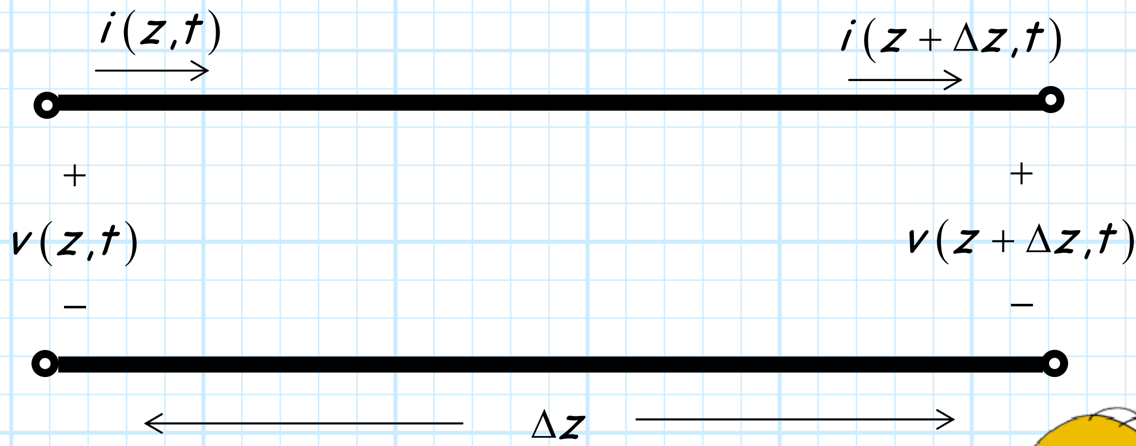
The Telegrapher Equations

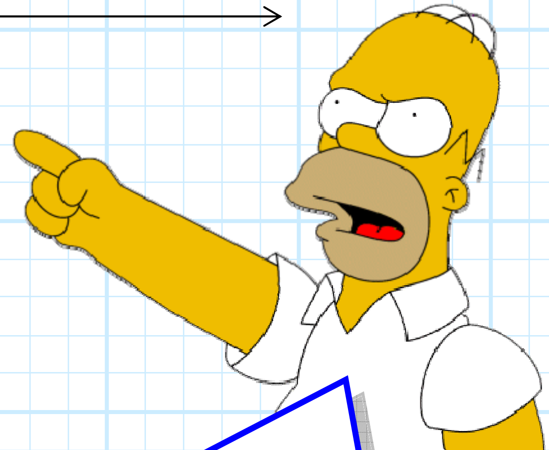
Consider a section of "wire":



Where:

$$i(z,t) \neq i(z+\Delta z,t)$$

$$v(z,t) \neq v(z+\Delta z,t)$$

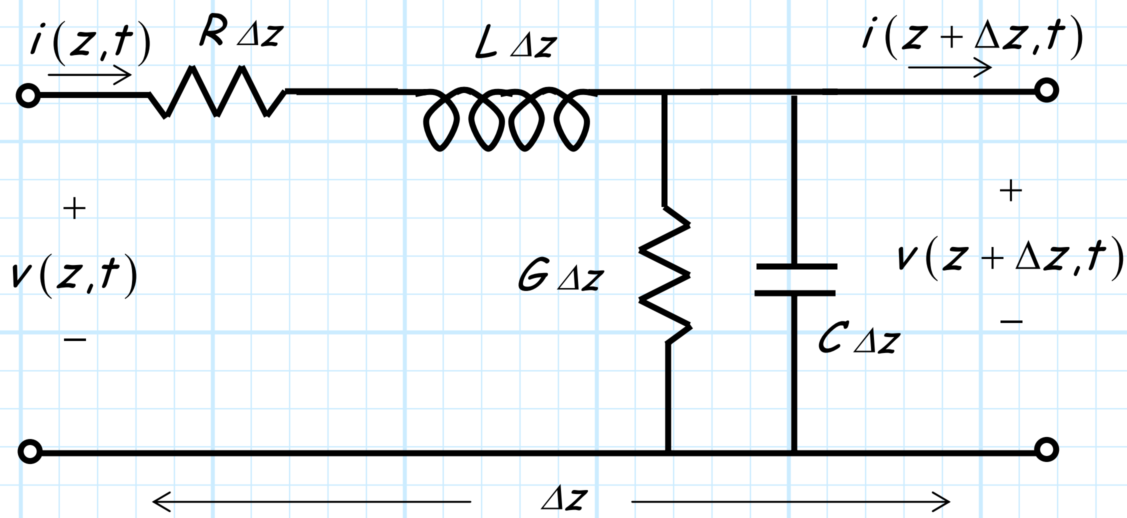


Q: *No way! Kirchoff's Laws tells me that:*

$$i(z,t) = i(z+\Delta z,t)$$
$$v(z,t) = v(z+\Delta z,t)$$

How can the voltage/current at the ***end*** *of the line (at* $z+\Delta z$ *) be* ***different*** *than the voltage/current at the* ***beginning*** *of the line (at z)??*

A: Way. The structure above actually exhibits some non-zero value of **inductance**, **capacitance**, **conductance**, and **admittance**! A more accurate transmission line model is therefore:



Where:

R = resistance/unit length
L = inductance/unit length
C = capacitance/unit length
G = conductance/unit length

$\therefore$ resistance of wire length Δz is $R\Delta z$

Now evaluating **KVL**, we find:

$$v(z+\Delta z,t)-v(z,t)=-R\Delta z\, i(z,t)-L\Delta z\frac{\partial i(z,t)}{\partial t}\neq 0$$

and from **KCL**:

$$i(z+\Delta z,t)-i(z,t)=-G\Delta z\, v(z,t)-C\Delta z\frac{\partial v(z,t)}{\partial t}\neq 0$$

Dividing the first equation by Δz, and then taking the limit as $\Delta z \rightarrow 0$:

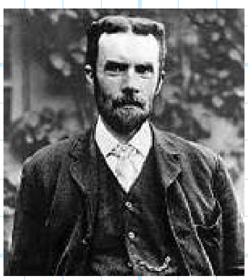
$$\lim_{\Delta z \to 0} \frac{v(z+\Delta z,t)-v(z,t)}{\Delta z} = -R\,i(z,t) - L\frac{\partial i(z,t)}{\partial t}$$

which, by definition of the derivative, becomes:

$$\frac{\partial v(z,t)}{\partial z} = -R\,i(z,t) - L\frac{\partial i(z,t)}{\partial t}$$

Similarly, the KCL equation becomes:

$$\frac{\partial i(z,t)}{\partial z} = -G\,v(z,t) - C\frac{\partial v(z,t)}{\partial t}$$



These coupled differential equations are quite famous! Derived by **Oliver Heavyside**, they are known as the **telegrapher's equations**, and are essentially the Maxwell's equations of transmission lines.

$$\frac{\partial v(z,t)}{\partial z} = -R\,i(z,t) - L\frac{\partial i(z,t)}{\partial t}$$

$$\frac{\partial i(z,t)}{\partial z} = -G\,v(z,t) - C\frac{\partial v(z,t)}{\partial t}$$

Although **mathematically** the functions $v(z,t)$ and current $i(z,t)$ can take any form, they can **physically** exist **only if** they satisfy the both of the differential equations shown above!